Math & Science Collaborative

Mathematics Standards

Grades 6-8

A Handbook for Parents

What are the mathematical goals for my child?

This handbook describes mathematics standards—expectations for students—for sixth, seventh and eighth grades. The goals and activities described here are based on Pennsylvania's state standards, on current approaches to teaching mathematics that are supported by research, and on recommendations from the National Council of Teachers of Mathematics.

The structure and teaching approaches of current curriculum programs are different from most parents' experiences with sixth-to-eighth grade mathematics. At all three grade levels, the mathematical topics students study include **numbers, geometry and measurement, algebra**, and **probability and data**. These topics are typically taught in units that are organized around the "big ideas," or key concepts, of mathematics. For example, a key concept in numbers is *ratio*. A key concept in geometry is *proportion*. A key concept in data is *sampling*—using results from a small group to predict what the results of a larger group would be.

Concepts are taught through problems that students could encounter in their daily lives. (See "What is problem-solving?" on page 10 for more information about the problem-solving approach). To build understanding of the concept, students first are assigned activities that ground the concept in reality. For example, before they learn ways to calculate the perimeters of irregular shapes, students might experiment with measuring shapes using pieces of string. Before learning how to gain information from a graph or a table, they may learn to gather and communicate data by designing their own surveys, interviewing classmates, interpreting the responses, and making their own tables and graphs to display the results. They study relationships, such as relationships between time and distance, or profit and cost, before writing statements using symbols—P [Profit] = I [Income] - C [Cost], for example.

In the process of solving problems and exploring concepts, students develop and refine a range of skills, including:

- **Communicating.** Students communicate the strategies they used to solve problems as well as their solutions. Using precise mathematical language, they organize, explain, define, describe, summarize, and defend their work.
- **Estimating.** Students estimate (make smart guesses) to help them predict and check their solutions. Among other strategies, they use "benchmarking"—estimating the size of a measurement or quantity by comparing it to a common measurement or quantity ("bigger than a right angle," "less than one-half"). Students learn to form "conjectures" (predictions based on reasoning) about what the solution will be—for example, whether the answer will be an odd or an even number. They visualize results—how big the area of a triangle will be if its perimeter is doubled, for example, or the effect of flipping or rotating a shape within a pattern.
- **Counting, calculating, and measuring.** Students understand the concept of sets of numbers, such as the set of whole numbers, or the range of numbers that can be plotted on a particular graph. They choose appropriate units to measure or count (degree, centimeter, ounce...). They determine whether to use mental math, paper and pencil, a calculator, or another kind of tool (graph paper, protractor, ruler...) to solve problems. They decide if they need to find a specific number or if an estimate is good enough.
- **Comparing.** Students compare numbers, shapes, and quantities to see whether they are equivalent (of equal value), proportionally the same (the same shape but not necessarily the same size), or subject to the same rate of change.
- **Checking.** Students monitor and revise their work as they go along, making sense of their solutions and checking the accuracy and effectiveness of their strategies. After solving problems, they reflect on the solutions. *Is the answer correct? If so, why is it correct?*
- **Representing.** Students "represent" (show) their work in many formats, including pictures, words, diagrams, graphs, charts, and tables. They "model" problems—laying out blocks to represent an

algebraic pattern, for example. They convert (change) one form into another—converting a fraction ($^3/_4$) into an equivalent decimal (.75), for example.

- **Extending.** Students consider their strategies and their solutions and ask themselves if they can apply what they've learned to another, similar problem. *How can I use what I learned about finding the area of a rectangle to find the area of a parallelogram? I created a coordinate graph to compare data about the arm span and height of my classmates—maybe that kind of graph will help me compare the costs and profit of products sold in the school store*. Finally, students formulate rules that can be used in all similar situations.

How are these goals accomplished in the classroom? Current approaches are designed to focus students on the hard work of learning, rather than on copying what their teachers do. For example, the teacher first introduces the task, providing a context for the problem and helping students see connections between the new task and previous work. Next, students solve the problem individually, in pairs, or in small groups. Finally, students share with the class their understanding of the task, their strategies for solving it, their solutions, and the formats they chose to communicate those solutions.

During these whole-group discussions, the teacher encourages all students to share, to question their classmates, and to ask their classmates to defend their statements. Such wide-ranging discussions help students see the task from different perspectives, learn to communicate mathematical ideas, move easily from one format to another (showing the same information in a table and on a graph, for example), and discover new strategies.

What is in this handbook?

In this handbook, you will find an outline of the standards for each grade level. "What is problem-solving?" on page 10 provides more information about the problem-solving approach. Pages 11 and 12 contain a sample problem for each grade level, with students' responses to the tasks. Page 13 offers suggested questions you can ask your child about homework and class work that can help stimulate his/her mathematical thinking, and a "student-friendly rubric"—a scoring guide written for students. The Glossary on page 14 provides information about mathematical terms and more extensive information about some mathematical concepts.

For more information

This handbook is only an outline. For more information, talk to your child's teacher or principal, or visit one of the following Web sites.

About curriculum and standards:

www.pde.pa.state.us (information about Pennsylvania's state standards)

www.illuminations.nctm.org (information about math standards and new teaching approaches)

www.math.msu.edu/cmp/index.html (information about the Connected Mathematics™ curriculum)

Math information, homework help, activities and games:

www.mathforum.org/dr.math (homework help, answers to math questions)

www.figurethis.org (activities geared toward families with students in grades 6-8)

See the Glossary for further explanations of mathematical terms and concepts.

Contents

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From Connected Mathematics: Problem 4.3: "Pirating Pizza" from *Bits & Pieces II*; Problem 3.1: "Mixing Juice," and Problem 3.3: "Sharing Pizza" from *Comparing and Scaling*; Problem 2.1: "Tiling Pools" from *Say It With Symbols*; Problem 19, page 43 "Extensions" from *Prime Time*; Problem 4.2: "Changing Speeds" from *Variables and Patterns* and *Teacher's Guide*; Problem 2, page 31 "Applications" from *Frogs, Fleas, and Painted Cubes*; Problem 4.1: "Fencing in Spaces" from *Covering and Surrounding*; Problem 3.1: "Discovering the Pythagorean Theorem" from *Looking for Pythagoras* © 1998 by Michigan State University, Glenda Lappan, James T. Fey, William M. Fitzgerald, Susan N. Friel, and Elizabeth D. Phillips. Published by Pearson Education, Inc., publishing as Pearson Prentice Hall. Used by permission.

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Sixth Grade Goals

Your sixth grade graduate should...

Numbers

Identify, analyze and compare "factors" (numbers that divide evenly into other numbers) and "multiples" (numbers that result from multiplying a given number with another number) of whole numbers, through:

- Analyzing games
- Creating tables
- Using "manipulatives" (objects)

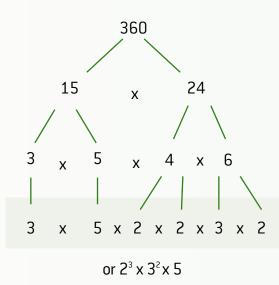
Recognize patterns and solve problems involving factors and multiples.

Find the "least common multiple"—the smallest number that is a multiple of two or more given numbers. For example, 6, 12, 18, and 24 are common multiples of 2 and 3, but 6 is the least common multiple.

Find the "greatest common factor"—the largest factor that two or more numbers have in common. For example, 1, 2, 3, and 6 are common factors of 18 and 24—6 is the greatest common factor.

Understand "prime number"* (a whole number greater than 1 with exactly two different factors, 1 and the number itself, such as 7) and "composite number"* (a whole number with more than two different factors, such as 14).

Factor tree



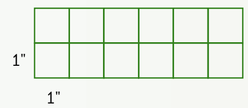
Prime factorization

Find the "prime factorization" of any composite number (expressing a number as the product of its prime factors).

Recognize that each whole number greater than 1 can be written as the product of its prime factors in only one way.

Use "exponents" to show how many times a factor is repeated ($2^3 \times 3^2 \times 5$—the prime factorization shown in the illustration).

Explore the relationship between "area" (the number of square units needed to cover a surface) and multiplication, using manipulatives (for example, arranging 12 square tiles to visualize the factors 1x12, 2x6, 3x4).



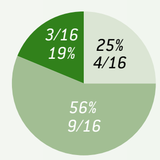
Tiles showing 2 x 6

Reason about the sums and products of odd and even numbers through numerical and geometric patterns.

Understand how fractions, decimals and percents are related, as parts of a whole.

Use skills and strategies developed for whole numbers to add, subtract, multiply and divide decimals and fractions.

Convert between fractions, decimals and percents ($\frac{5}{8}$=.625=62.5%=62$\frac{1}{2}$%) and show a given quantity as a fraction, a decimal, or a percent, in multiple ways (pictures, graphs, numbers...).



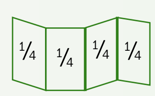
Use a fraction or a decimal to represent a given situation—for example, a slice of pie as $\frac{1}{8}$, a quarter inch on a ruler, a dime as .10.

Find a percent of a given number. Know that percent means "out of 100."

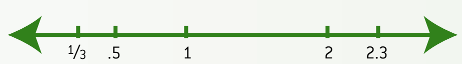
Use percents in realistic situations, such as estimating a tip, a tax, or a discount.

Compare fractions and decimals, using strategies such as:

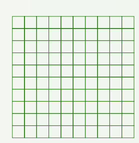
- Fraction strips (paper strips divided into equal marked parts)



- Number line



- Grid with 100 squares



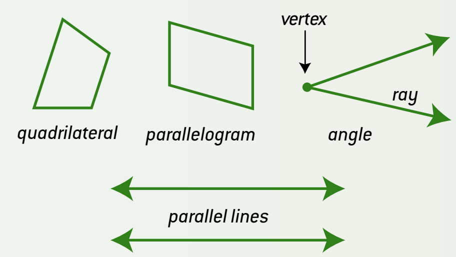
- "Benchmarks" (estimating by comparing uncommon fractions to common fractions, such as $\frac{1}{2}$—for example, $\frac{7}{32}$ is close to $\frac{8}{32}$, or $\frac{1}{4}$)
- Rewriting the numbers in a common form (with common denominators or numerators, or as decimals or percents)

Geometry and Measurement

Recognize and name the dimensions of 2-dimensional shapes: height, length, width, base, diameter, radius.

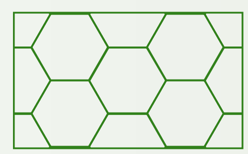
Recognize, name, draw, compare, and know the properties (defining characteristics) of:

- Polygons (2-dimensional figures made of line segments)
- Triangles
- Quadrilaterals (4-sided polygons)
- Parallelograms
- Parallel lines
- Angles
- Vertices (corners where two rays form angles—plural form of *vertex*)
- Circles



Explore the properties of geometric shapes in realistic situations, such as the design of a building or the patterns in a natural form (a honeycomb, a shell...).

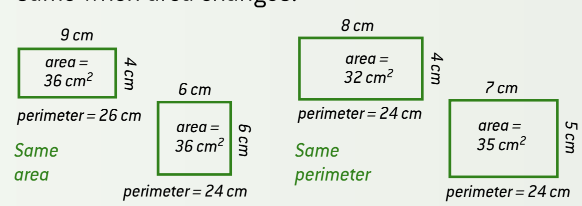
Analyze shapes to determine whether they "tessellate" (fit together to cover a flat surface with no overlapping edges).



Tessellation

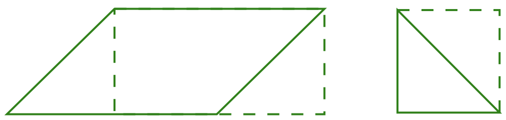
Experiment with finding area and "perimeter" (the sum of the lengths of the sides) for 2-dimensional shapes by measuring, estimating (measuring irregular shapes with a string, for example), and calculating.

Understand and show how area can be the same when perimeter changes and perimeter can be the same when area changes.



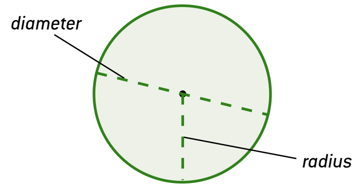
** See the Glossary for more information.*

Understand how the areas of triangles and parallelograms are related to the areas of rectangles.



Create rules for finding area and perimeter after exploring and analyzing shapes and measurements.

Measure the diameter, the radius, the circumference (the distance around a circle), and the area of a circle, and explain the relationships between them.



Experiment to find the approximate value of "pi" (a number that expresses the relationship between circumference and diameter) by measuring circles of different sizes. Calculate the area and circumference of circles using pi.

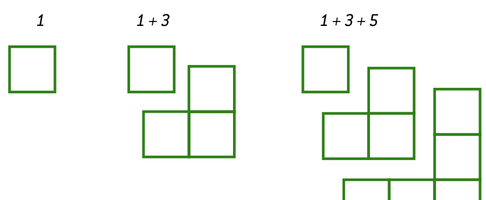
Identify angles, estimate and measure the degrees of angles, and understand what the measure of an angle represents (for example, a right angle on an envelope and a right angle on a skyscraper are both 90°).

Estimate angle measures using benchmarks (comparing an unknown angle to an easily recognizable angle, such as a right angle).

When measuring, choose appropriate tools (ruler, angle ruler, protractor, compass, tape measure...) and appropriate units (inch, centimeter, degree...).

Algebra

Analyze, create, continue, and discover rules for patterns of numbers and geometric figures (see illustration).



A number pattern

Model problem-solving situations using numbers, diagrams, pictures or objects.

Probability and Data

Explore the concept of probability through experimenting (flipping coins, using spinners, rolling number cubes) and through mathematical reasoning.

Determine the number of possible outcomes for an event, through observations, experiments, and mathematical models. Compare "experimental probability" to "theoretical probability." For example, roll a pair of number cubes and add together the numbers shown. Repeat this 20 times (experimental probability). How many times did you get an even number? Calculate all the possibilities (theoretical probability). Compare results. How would your numbers compare if you rolled the number cubes 400 times?

Understand the concept of "equally likely" outcomes.

Express probability using fractions, decimals and percents.

Make predictions based on probability. ("A spinner has three equal sections: one red, one yellow, one blue. What are your chances of the pointer landing on yellow?")

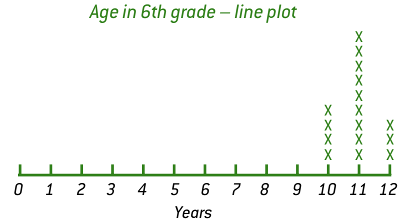
Evaluate statements about probability. Explore probability in realistic situations, including genetic traits (such as eye color) and advertising promotions.

Develop strategies for collecting and organizing different kinds of information—"categorical" (information that can't be expressed in numbers, such as your favorite ice cream flavor) and "numerical" (information that *can* be expressed in numbers, such as how many flavors of ice cream you've tried).

Pose questions ("How many letters are there in the names of everyone in our class?"). Collect, analyze, and interpret information (data) to answer those questions.

Create, read and interpret line plots, tables, bar graphs, stem and leaf plots (see below), and circle graphs.

Age in 6th grade – line plot

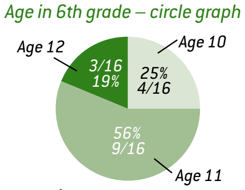


Temperature ranges November – January

Stem	Leaf
2	1, 2, 3, 3, 4, 5, 6, 7, 9
3	1, 2, 3, 3, 4, 5, 5, 6
4	1, 2, 3, 5

Stem and leaf plot (stems represent tens, leaves represent ones)
Key: 3|5 means 35°

Age in 6th grade – circle graph



Determine which format is the best way to display data for a given situation.

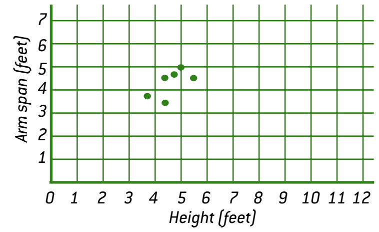
Explore the idea of "average" (what is "typical") by finding:

- The "mean" (the average, in common usage)
- The "median" (the number that would fall in the middle if all the numbers were listed in order)
- The "mode" (the number that occurs most often)

Explore other related data concepts by finding:

- The largest and smallest quantities in a data set ("Javon has tried 11 flavors of ice cream. Mindy has tried five.")
- The "range" (the difference between the largest and smallest quantities)
- "Outliers" (unusual data—for example, "Brandy has tried 60 ice cream flavors. Her mom owns an ice cream store.")

Recognize relationships shown in graphs, such as the relationship between arm span and height shown in the illustration.



Compare data sets ("How does our 'ice cream flavor' data compare to the second period class?").

Make reasonable decisions or judgments based on sets of data.

Problem-solving and Mathematical Thinking

Estimate and predict using logic and reasoning.

Choose or develop strategies and procedures for solving problems with whole numbers, fractions, decimals, percents, and geometric shapes.

After exploring a range of similar problems, define rules that can be used in all similar situations. (For example, measure the circumference and the diameter of a variety of circles. Compare their relationships, and define a rule that applies to the relationship between circumference and diameter in all circles).

Evaluate and interpret results. (Does the answer seem correct? Why or why not?)

Communicate results with numbers, charts, graphs, diagrams and/or pictures, formulas, and in words.

Seventh Grade Goals

Your seventh grade graduate should…

Numbers

Use "integers" (whole numbers and their opposites) in realistic situations, such as degrees on a thermometer or points won or lost in a game.

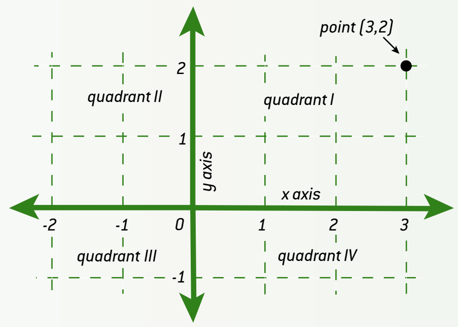
Locate integers on a number line.

Compare integers (-261 < -50).

Define rules for adding, subtracting, multiplying and dividing integers (such as "multiplying two negatives produces a positive"), by:

- Developing strategies
- Analyzing patterns
- Using models

Create graphs that include all four quadrants (see illustration), by hand and with a graphing calculator (a special calculator that can create graphs).



Develop ways to compare quantities (other than subtraction), including:

- Ratios ("the ratio of boys to girls in our class is 2 to 3")
- Fractions ("$^{1}/_{5}$ of the girls in the class play on a sports team")
- Decimals (".25 of the boys in the class play on a sports team")
- Percents ("5% of the students in the class speak English as a second language")
- Unit rates (a type of ratio—determining the cost per item to find the better deal, for example)
- Scaling up or down (scaling up a recipe that serves 4 to serve 6, for example)

Evaluate statements about quantities. ("Buy three for $5.00." "Buy seven for $9.00." Which is the better buy?)

Use strategies for comparing quantities and estimating proportions in realistic situations—for example, looking at a photograph of a crowd and counting the number of people in a small area, then scaling up to estimate the total.

Use unit rates to solve problems. ("If Quentin's car gets 22 miles per gallon, how many miles can he travel if he has a full 20-gallon tank?")

Become familiar with using very large numbers and very small numbers.

Choose units of measure that make sense—for example, about 2 tons rather than 4,137 pounds.

Find "benchmarks" (reference points, such as the number of people that would fill a stadium, the length of 20 football fields…) that can give meaning to large numbers.

Read, write, and rank large and small numbers in the context of realistic situations, using "standard notation" (4,300,000 or .00510) and "scientific notation" ($4.3x10^{6}$ or 5.1×10^{-3}).

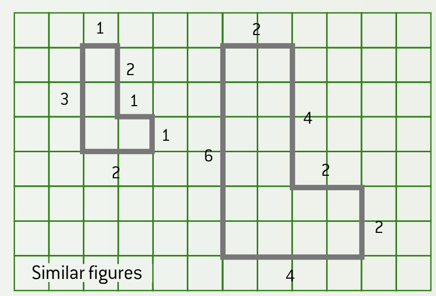
Compare large quantities and large measurements using subtraction, ratios and rates.

Use estimates and rounded numbers to describe and compare quantities.

Evaluate statements about large numbers—for example, statements in news reports.

Geometry and Measurement

Develop an understanding of "similarity" (when figures have the same shape, the same angles, and all sides are proportional but not necessarily the same size). Use the concept of similarity to solve everyday problems.



Enlarge and reduce shapes "to scale" (changing them in a fixed ratio to their original size) by:

- Using graph paper units (see illustration above)
- Multiplying side lengths by "scale factors" (the ratio used to enlarge or reduce similar figures—for example, if one figure is $^{1}/_{4}$ the size of the other, the

scale factor is $\frac{1}{4}$, 25%, or .25; if another figure is 3 times bigger, the scale factor is 3 or 300%)

- Defining rules for using scale factors (for example, in the illustration on page 6, the scale factor to enlarge the smaller figure to the size of the larger figure is 2)

Compare:

- The "area" (the measure of the surface of a 2-dimensional shape in square units) of similar figures—see illustration on page 6
- The ratios of lengths and areas for similar figures (sides compared to sides, area compared to area)
- Angles in similar figures

Understand similarity in terms of "equivalence" (equal value)—equivalent ratios (1:5, 2:10) and equivalent fractions ($\frac{1}{5}$, $\frac{2}{10}$).

Use knowledge of similarity to find the measurements of real objects and drawn shapes (see illustration).

Experiment with "manipulatives" (arranging cubes in layers, folding flat patterns into boxes and cylinders...) to develop an understanding of surface area and volume.

Use strategies developed for measuring the area of 2-dimensional shapes to find the volume of 3-dimensional shapes.

Compare the volumes of different shapes—cubes, rectangular prisms, cylinders, cones and spheres.

Show how changing one or more of the dimensions of a 3-dimensional shape changes its volume.

Algebra

Understand the concept of "variables"—quantities that can change, such as the amount of time it takes to complete a task or the number of items sold in a store on different days.

Recognize relationships between variables, such as time, rate and distance (distance traveled is related to rate of speed and elapsed time).

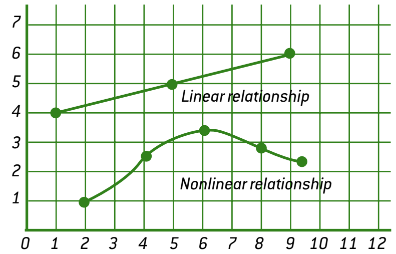
Identify the "independent variable" (the variable that stands alone, such as time), and the "dependent variable" (the variable that is influenced by the other variable, such as distance traveled).

Understand "proportional relationships"—when two variables change by the same factor. (For example, 1 item sold produces $2.00 in profit, so 3 items sold produce $6.00 in profit, 10 items sold produce $20.00 in profit....)

Find and predict patterns in data (for example, if 4 items were sold and $8.00 profit was made on the sale, how much profit would there be if 24 items were sold?).

Understand how a situation can be represented on a graph.

Recognize that a "linear relationship" (when the rate of change is constant) on a graph appears as a straight line, rather than a curved or fluctuating line (see illustration).

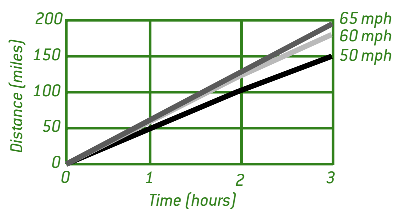


Understand that changes in rate change the "slope" (the incline, or steepness) of a line on a graph. ("How long would it take to travel between two cities if we are traveling 50 miles per hour, 60 miles per hour, or 65 miles per hour?" See the graph and table below.)

Create tables, graphs, and verbal descriptions that effectively show how variables are related. Plot information on coordinate graphs; complete tables with information.

Table and graph showing distance traveled at different rates of speed

Time (hours)	50 mph	60 mph	65 mph
0	0	0	0
1	50	60	65
2	100	120	130
3	150	180	195
n (other numbers)	50n	60n	65n



Create "symbolic rules" (statements that use symbols to express a mathematical rule) to show relationships between variables. For example, *d* (distance) = 10*t* (time, in hours).

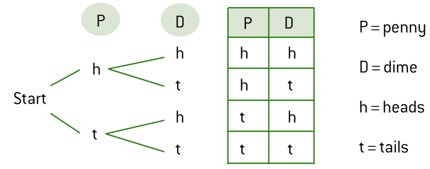
Show and describe the same information in a table, a graph, a symbolic rule, and words, and describe the advantages and disadvantages of each format. Use a graphing calculator to convert a symbolic rule into graphs and tables.

Probability and Data

Understand probability as expected outcomes over time, or over many trials.

Develop strategies to find the number of possible outcomes for events, including:

- Making a list or a chart
- Using a "counting tree" (see illustration) when outcomes are equally likely



Counting tree for penny and dime toss

- Using an "area model" (mapping possible outcomes on a grid, such as graph paper), when outcomes are not equally likely

Show probabilities as fractions, decimals, and percents (for example, the chance of a coin toss resulting in "tails" is 50%, .50, or $\frac{1}{2}$). Show probabilities on a graph and in a table.

Analyze situations where probability is affected by "dependent events" (events that depend on the outcome of an earlier event) and "independent events" (events that don't depend on another event's outcome). For example, the number of possible paths through a maze changes with each choice a traveler makes.

Explore long-term averages, or "expected value," in realistic situations, such as a basketball player's chances of scoring points given her scoring average, or the profitability of a carnival game.

Problem-solving and Mathematical Thinking

Estimate and predict using logic and reasoning.

Choose or invent strategies and procedures for solving problems with integers, fractions, decimals, percents, and geometric shapes.

After exploring a range of similar problems, define rules that can be used in all similar situations. (For example, after solving a number of problems involving the surface area and volume of certain 3-dimensional shapes, define rules that apply to other 3-dimensional shapes.)

Evaluate and interpret results. (Does the answer seem correct? Why or why not?)

Communicate results with numbers, charts, graphs, diagrams and/or pictures, formulas, and in words.

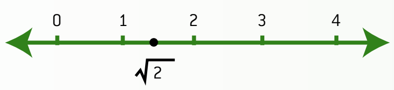
Eighth Grade Goals

Your eighth grade graduate should...

Numbers

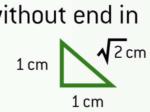
Explore “rational numbers” * and related concepts.

- Explore “repeating decimals” (decimals that continue without end in a repeating pattern, such as .333... or .191919....) and “terminating decimals” (decimals that don’t continue, such as .25). Identify repeating and terminating decimals as rational numbers.
- Distinguish between rational numbers and “irrational numbers.” *
- Find the estimated location of irrational numbers on a number line.



(the square root of 2, or 1.414213562...)*

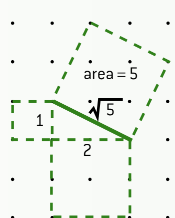
- Understand how irrational numbers can be represented geometrically and as “nonrepeating decimals” (decimals that continue without end in no particular pattern, such as 1.414213562...).



Geometry and Measurement

Experiment to find ways of measuring figures that have side lengths that are irrational numbers.

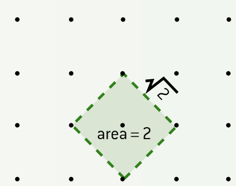
- Find the distance between two points on a dot grid (see illustration).



Finding the distance between 2 points (C):

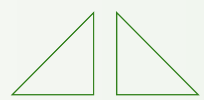
$(1)^2 + (2)^2 = C^2$, so $1 + 4 = C^2$, so $5 = C^2$ or $\sqrt{5} = C$

- Measure the “slope” (the incline, or steepness) of line segments shown on a dot grid.
- Find the area of figures drawn on a dot grid.
- Find the measurement of side lengths of squares by finding the area of the square and knowing that each side is the square root of that area.



Experiment to find ways of measuring side lengths that are unknown. For example, use the “Pythagorean Theorem.” *

Recognize shapes that are “congruent” (the same size and shape—see illustration).



Congruent shapes

Recognize symmetry in shapes and designs, including:

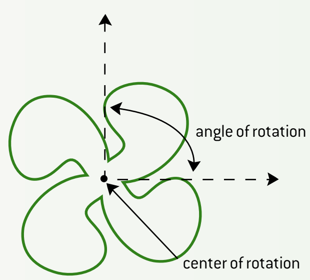
- Mirror-image or “reflectional” symmetry (such as a butterfly’s wings)
- “Rotational” symmetry (shapes that would match exactly if they were rotated, such as a windmill’s blades)
- “Translational” symmetry (shapes repeated in a pattern, such as a design in fabric or wallpaper)

Identify:

- “Lines of symmetry” (the line or lines that can divide a shape into mirror images)
- “Angles of rotation” (in a design created around a center point, the degree of rotation needed to make shapes match—see illustration below)

Draw symmetrical shapes and patterns using tracing paper, mirrors, folded paper, rulers, and angle rulers or protractors.

Write specific mathematical directions for “transforming” shapes (making mirror-image copies or rotated copies, for example).



Explore the effects of transforming a shape, then transforming the copy.

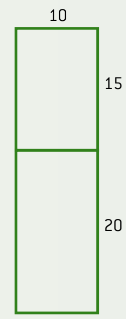
Algebra

Create and make sense of “symbolic sentences” (equations that use symbols such as x and y to represent unknown numbers) involving addition, subtraction, multiplication, division, and exponents (y^2). (*Equation:* statement with an equal sign showing two quantities are equal, such as $25=5x$.)

Use the “order of operations” (rules about which operation to perform first in problems that require more than one operation—both addition and multiplication, for example) and apply them to everyday problems.

Recognize equivalent equations and expressions (numbers or expressions that are of equal value) and show that they are equivalent using algebraic properties such as:

- The “Commutative Property:” numbers being multiplied or added can be switched around without changing the result ($a \times b = b \times a$)
- The “Distributive Property:” solving problems using the relationship between addition and multiplication, or subtraction and multiplication—for example, $a(b + c)$ is the same as $(a \times b) + (a \times c)$ (see illustration)



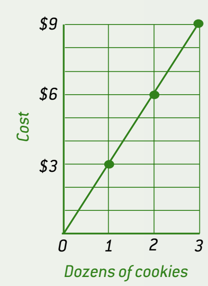
Finding the total area using the Distributive Property: Multiply the width by the entire length: $10(15 + 20)=350$, or find the areas of each rectangle and add them: $10(15) + 10(20)=350$

Show whether or not equations are equivalent by graphing them to see if the graphs are the same.

Find the slope given the “coordinates” (the points on a graph) of two points.

Explore:

- “Linear” relationships (when there is a constant rate of change between two variables—for example, for every dozen cookies sold at the fair, the total cost increases by \$3.00) (*Variable:* a quantity that can change.)



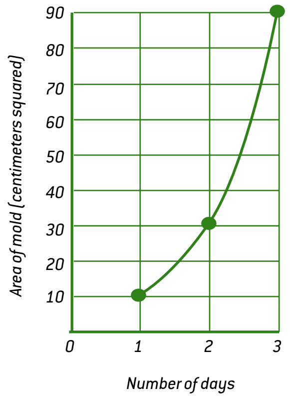
- “Inverse” relationships (relationships characterized by one variable increasing as the other decreases)
- “Nonlinear” relationships (when there isn’t a constant rate of change between two variables—for example, money in a savings account earns a different amount of interest each year, as the interest on the original amount earns interest)

Recognize patterns that show “exponential change”—

* See the Glossary for more information.

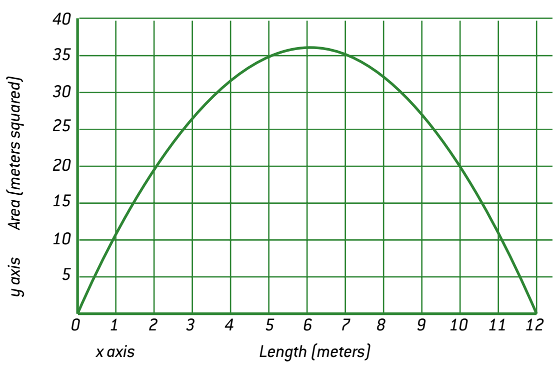
increasing an original amount by *repeatedly* doubling it (2, 4, 8, 16, 32…), or tripling it, or quadrupling it, and so on, or decreasing an original amount by repeatedly halving it ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$…), or reducing it by one-third, and so on.

Find "growth factors"—the rate of exponential growth. For example, if mold growing on bread triples its size every day, the growth factor would be 3.



Find "decay factors"—the rate of exponential decay (decrease).

Explore "quadratic relationships." * (Example: "Shane wants to build a rectangular pen for his dog. He has 24 meters of fencing. What are the different size pens Shane can build? Which would provide the most area for Shane's dog?" See the graph and table below.)



Graph showing the maximum area for Shane's dog pen

Length (l)	Width (12-l)	Perimeter 2(l) +2(12-l)	Area l(12-l)	
1	11	24 m	11 m^2	
2	10	24 m	20 m^2	
3	9	24 m	27 m^2	
4	8	24 m	32 m^2	
5	7	24 m	35 m^2	
6	6	24 m	36 m^2	maximum area
7	5	24 m	35 m^2	
8	4	24 m	32 m^2	
9	3	24 m	27 m^2	
10	2	24 m	20 m^2	
11	1	24 m	11 m^2	

Table showing possible areas for Shane's dog pen

Compare quadratic relationships to linear and exponential relationships.

Identify on a graph of a quadratic relationship:

- The "line of symmetry" (the line that divides the two sides of the graph into mirror images)
- The "intercepts" (the points where the graph crosses the x-axis or y-axis)
- The "vertex" (the point where the line of symmetry passes through the graph)
- The "maximum value" (the greatest y value for an equation), or
- The "minimum value" (the smallest y value for an equation)

Find ways of showing linear, exponential, and quadratic relationships.

- Create tables.
- Draw lines/curves that reflect the trend of the data.
- Write equations ($y = 3x + 5$).
- Plot data on a coordinate grid.
- Create graphs by hand and with a graphing calculator (a special calculator that can create graphs).
- Show how changes in conditions would change the line of a graph.
- Verbally describe the information shown on a graph.

Use tables, graphs, and equations to solve problems involving linear, exponential, and quadratic relationships.

- Recognize patterns in tables.
- Recognize the shapes of their graphs.
- Identify equivalent expressions and equations.

Data

Explore "sampling" as a method of gathering information (using results from a small group to predict what the results of a larger group would be—for example, polling a sample of likely voters about their choices in an election), including:

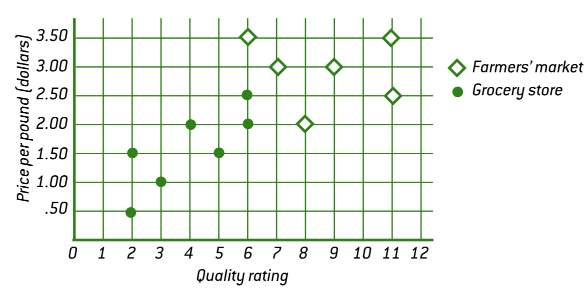
- "Representative" samples (such as selecting a sample that includes Democrats and Republicans, living in urban, suburban and rural areas)
- "Random" samples (a kind of representative sample where everyone in the large group has an equally likely chance of being selected for the sample group)

Compare methods of sampling and consider their advantages and disadvantages. ("How does the size of our sample affect the accuracy of our information? Might our sample be 'biased'—skewed in one direction or the other—for any other reason?")

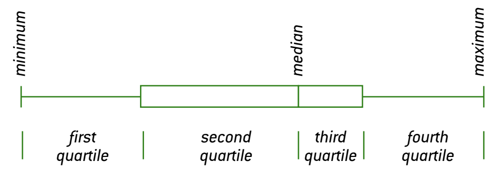
Evaluate questions on surveys to determine whether they can produce the desired information.

Compare sets of data using:

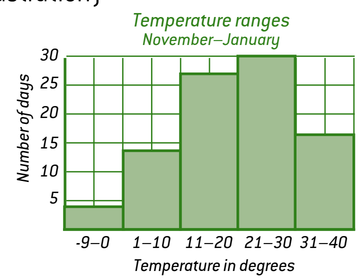
- "Scatter plots"—graphs that can show comparable information for two variables, such as the price and quality rating of a group of products (see illustration)



- "Box-and-whisker plots"—a method for grouping ordered data (see illustration) to show quartiles (data divided into four parts), the median (middle point of the data), the maximum value (largest number), and the minimum value (smallest number)



- "Histograms"—graphs that can show ranges of data, such as how many days the temperature was in the 20s, how many days it was in the 30s… (see illustration)



Problem-solving and Mathematical Thinking

Estimate and predict using logic and reasoning.

Choose or invent strategies and procedures for solving problems with integers, fractions, decimals, percents, and geometric shapes.

After exploring a range of similar problems, define rules that can be used in all similar situations. (For example, define a rule for finding the side of any right triangle.)

Evaluate and interpret results. (Does the answer seem correct? Why or why not?)

Organize and communicate results with numbers, charts, graphs, diagrams and/or pictures, formulas, and by verbal and written explanations.

Helping your child develop problem-solving skills

In the classroom, the teacher's goal is to create an environment in which students feel comfortable questioning themselves and their classmates, and sometimes working through frustration and confusion—all part of the learning process. At home, parents can provide a similar kind of support by asking questions that push their child's learning further.

Questions you might ask while your child is working on a problem:

- "What are you being asked to find out?"
- "What does the problem tell you?"
- "Is there anything you don't understand? Where can you find answers to your questions?"
- "Will it help to make a chart, a list, a drawing, or a diagram?"
- "What will you try first? ...Next?"
- "What do you estimate the answer will be?"
- "Is your strategy working? Why or why not?"
- "How do you know if your answer is right or wrong?"
- "Let's look at the problem again. Did you answer the question?"

Questions you might ask about your child's completed work:

- "What different ways did you try to solve the problem?"
- "What worked or didn't work?"
- "How did you know?"
- "Do you know another way to solve this?"
- "Do you know another way to show your answer (a chart, a graph, a statement with symbols...)?"

A "student-friendly" rubric

One way to assess whether or not a student's work meets standards is to use a "rubric," or scoring guide. A rubric shows students what the teacher expects. Below is a general rubric for a mathematics problem for grades 6-8, developed by Pittsburgh Public Schools' PRIME (Pittsburgh Reform In Mathematics Education) team.

4 "Advanced"

My answer shows that I really understand the mathematics in the problem.

My answer is right.

I included *all* calculations—charts, tables and/or graphs, drawings or diagrams—that show I understand the mathematics in the problem.

My written description is clear, labeled and well organized.

A reader can *clearly* follow my ideas and thinking.

3 "Proficient"

My answer shows that I understand the *most* important mathematics in the problem.

My answer might be wrong because of a small mistake.

I have included *all* of my calculations—a chart, table, and/or graph, or a drawing or diagram—that show I understand the mathematics in the problem.

Most of my written description is clear and well organized.

A reader can follow *most* of my ideas and thinking.

2 "Basic"

My answer shows that I understand *some* of the mathematics in the problem but I may have left out some important mathematics or I may have included incorrect mathematics.

My answer is wrong. (My answer could be right, but there is no supporting work.)

I included *some* calculations—a chart, table or graph, a drawing or diagram—but they are unfinished or have mistakes.

Some of my written description is clear and organized.

A reader *may* have a hard time following my thinking.

1 "Below Basic"

My answer shows that I do not understand *any* of the mathematics in the problem.

My answer is incorrect or I used the wrong mathematical approach.

I may have included *some* calculations—a chart, table, or graph, a drawing or diagram—but they *cannot be used* to solve this problem.

If I have a written description, it is not organized *or* it is incomplete *or* it is unclear.

A reader would have a hard time following my thinking.

0

I have not tried to solve the problem. There is no written evidence to show that I thought about the problem.

Glossary

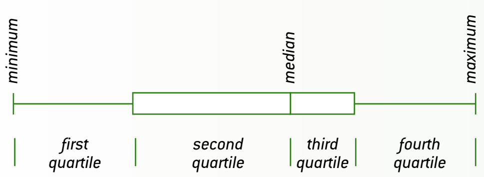
This Glossary is intended to provide information for parent readers. It is not intended to provide complete, technical definitions of mathematical terms.

algebraic expression: A variable—or any combination of numbers, variables, and symbols that have been combined using algebraic operations—showing a mathematical relationship ($3n \times 7$, $4a - 9$).

area: The measure, in square units, of the surface of a two-dimensional shape.

benchmark, benchmarking: A common measurement or quantity used as a reference point to estimate the size of an unknown measurement or quantity. For example, a right angle can be a benchmark—other angles are smaller or larger than a right angle. This estimation strategy is called *benchmarking*.

box-and-whisker plot: A graphic method for grouping ordered data to show how the data are distributed. For example, a box-and-whisker plot could be used to show how many people under the age of 50 live in a certain area, and whether the majority of those people are between 35 and 49, or 25 and 30, and so on. Data are grouped into four sections called "quartiles." A box is drawn to include the middle 50% of the data items and whiskers are extended to the left and right of the box showing the maximum and minimum values (largest and smallest numbers). Any data items that are located beyond the reach of the whiskers are called "outliers" and are indicated with a star or other symbol. The median number (the middle point of the data) is shown as a vertical line inside the box.

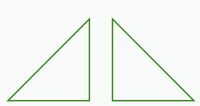


box-and-whisker plot

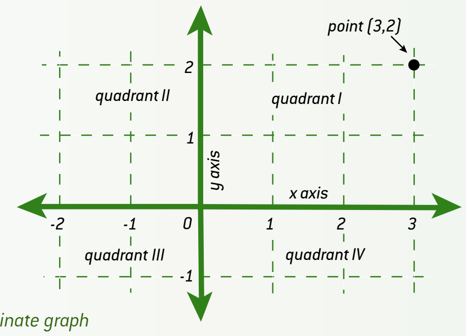
circumference: The measure of the distance around a circle.

composite number: A whole number with more than two different whole-number factors, such as the number 14. (The number 1 is neither prime nor composite.)

congruent, congruence: Having the same size and shape, although possibly in different positions. Such shapes have *congruence*.


congruent shapes

coordinate graph: A two-dimensional graph made up of a horizontal x axis and a vertical y axis and ordered pairs—such as (3,2) in the illustration—representing values. Coordinate graphs are used to show the relationship between two variables—for example, length and width, profit and cost, distance traveled and time.



coordinate graph

data: Information, in the form of facts or figures, gathered to answer questions, such as counts, ratings, measurements, and opinions. Data are either categorical or numerical. *Numerical data* are in the form of numbers (number of children in families, height, pulse rate...) while *categorical data* represent information expressed as words (names of students in the class, favorite color, kinds of pets...).

dependent variable: The variable that is influenced by the other variable. For example, if Jill traveled in a car at 55 mph and the variable *D* equals "the distance Jill has traveled (measured in miles)" and the variable *T* equals "the time Jill has been traveling (measured in hours)," then *D* is a function of *T*. In other words, *D* (distance) is determined by *T*(time)—distance is the dependent variable. On a coordinate graph, the dependent variable is plotted on the y axis.

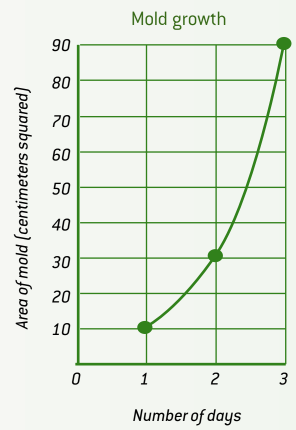
diameter: Any line segment that passes through the center of the circle, has endpoints on the circle, and divides the circle in half.

equation: A statement showing that two mathematical expressions are equal, such as $2+2=4$ or $5y=10$.

equivalent: Of equal value. For example, $\frac{3}{4}$, .75, and 75% are equivalent.

exponent: A number written to the right and above another number—the "base" number—that shows how many times the base number is to be multiplied by itself. For example, 4^3 means 4x4x4. "4" is the base and "3" is the exponent.

exponential relationship: A pattern of increase or decrease (growth or decay) in which a quantity is scaled up or down by the same factor at each step. For example, it might be repeatedly doubled or multiplied by 2 (2, 4, 8, 16, 32...), or tripled (1, 3, 9, 27...), or quadrupled, and so on; or repeatedly halved or divided by 2 ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$...), or reduced by one-third, and so on.



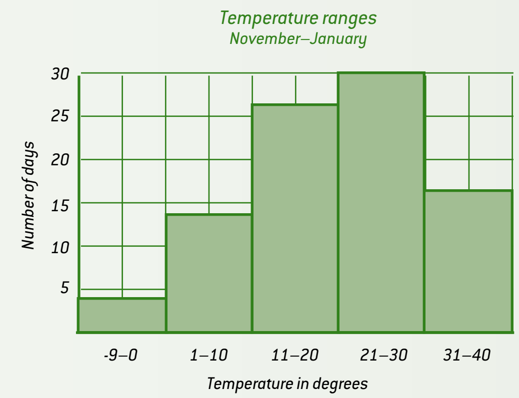
an exponential relationship

factor: A number that divides into another number without leaving a remainder. For example, 1, 3, 5, and 15 are factors of 15 because all of those numbers divide evenly into 15.

function: A set of ordered pairs of numbers, usually related by a rule. Functions are used to describe relationships between variables. For example, the cost of parking in a garage is a function of the time spent parked in the garage.

grid: Any pattern of crisscrossing lines that creates squares, such as the pattern on graph paper.

histogram: A kind of bar graph used to show ranges of data. Instead of representing one number, each bar represents a range of numbers.



independent variable: The variable that is not influenced by the other variable. For example, if Jill traveled in a car at 55 mph and the variable *D* equals "the distance Jill has traveled (measured in miles)" and the variable *T* equals "the time Jill has been traveling (measured in hours)," then *D* is a function of *T*. In other words, *D* (distance) is determined by *T*(time)—time is the independent variable. On a coordinate graph, the independent variable is plotted on the x axis.

integer: The set of numbers that includes all whole numbers and their opposites (negative numbers). For example, -5, 2, and 203 are integers, but $\frac{1}{2}$, .523, and -20.5 are not integers.

irrational number: A number that can't be written as a fraction (or ratio) of two integers. Irrational numbers are decimal numbers that never end or repeat. For example, pi (3.14285713...) and the square roots of some numbers that are not perfect squares ($\sqrt{2}$ and $\sqrt{5}$) are irrational numbers.

linear relationship: A relationship between two variables that change at a constant rate. For example, the distance traveled by a car going at 45 miles per hour, and the number of hours traveled, have a linear relationship. Shown as an equation, this relationship can be expressed as d (distance) = $45t$ (time, in hours, multiplied by the rat e: 45). This is a *linear equation*.

manipulative: An object used to help students visualize a problem or concept. Manipulatives may be blocks, cubes, flat shapes such as tiles, straws, counters....

multiple: The product of a whole number and another whole number. For example, the multiples of 4 include 4 (4x1), 8 (4x2), 12 (4x3), 16 (4x4)....

***n*:** A letter commonly used to represent an unknown or changing number.

nonlinear relationship: A relationship between two variables that don't change at a constant rate. For example, the graph illustrating "exponential relationship" at left shows a nonlinear relationship between the area of mold growth and the number of days passing.

nonrepeating decimal: A decimal number that continues without end in no particular pattern.

numerical expression: A combination of numbers and operation sign(s) that represent a mathematical relationship. For example, 5x5 or 24x2+5.

operation: A process or action performed on a number or numbers. Addition, subtraction, multiplication and division are examples of operations.

order of operations: Rules that prescribe which operation to perform first, in a problem that involves more than one operation. For example, $3 + (5+3)^2 \times 6$.

1. Carry out operations inside parentheses and other grouping symbols, such as brackets or root signs. (In the example, 5+3=8).
2. Simplify exponents ($8^2 = 64$).
3. Multiply or divide in order from left to right (64x6=384).
4. Add and subtract in order from left to right (3+384=387).

parallelogram: A four-sided figure (a quadrilateral) with opposite sides that are equal and parallel.


parallelogram

perimeter: The measure of the distance around a figure.

pi: A number that expresses the relationship between the circumference and diameter of any circle (the ratio of the circumference divided by the diameter). Pi is an irrational number usually written as 3.141592… or as π .

polygon: A 2-dimensional, closed, flat shape made up of three or more connected line segments that don't cross over each other.

prime factors: Factors that can't be divided evenly into smaller whole numbers. For example, 2 and 3 are prime factors of 12.

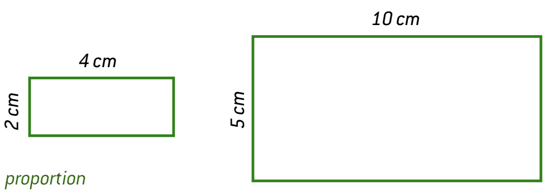
prime factorization: Showing a whole number greater than 1 as the product of its prime factors. For example, the prime factorization of 12 is 2x2x3. Each whole number can be written as the product of its prime factors in only one way.

prime number: A whole number greater than 1 with exactly two different factors, 1 and the number itself, such as the number 7. (The number 1 is neither prime nor composite.)

probability: A number that indicates the relative likelihood that an event will happen. *Experimental probability* is determined through experimentation. For example, if a basketball player succeeded in making 20 baskets out of 80 free throws, her experimental probability of making a basket would be 20 out of 80, or 25%. *Theoretical probability* is found by analyzing the possibilities. Because we know that there are two possible outcomes for a coin toss and one of those is tails, we calculate the probability of getting tails to be 1 out of 2 or 50%.

product: The number that results from multiplying two numbers. For example, 12 is the product of 6 and 2.

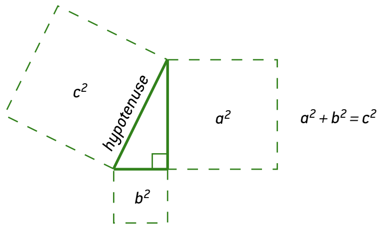
proportion: An equation showing that two ratios are equal. For example, the rectangles in the illustration are proportionally the same because 2:4 is equal to 5:10.



proportion

Pythagorean Theorem: A rule for finding the length of one side of a right triangle when the lengths of the two other sides are given.

The sum of the areas of the squares on the legs of a right triangle is equal to the area of the square on the hypotenuse: $a^2 + b^2 = c^2$.



quadratic relationship: A relationship between variables in which one variable is raised to the second power (in other words, "squared"—with the exponent ^{2}). For example, $x^2 + 2x + 1 = y$. Quadratic equations are used in situations where one variable changes with respect to the square of the other variable, such as area (see the example on page 9).

quartiles: Ordered data divided into four parts. (See "box-and-whisker plot.")

radius: The distance from the center of a circle to its edge.

ratio: A comparison between two quantities that shows the size of one relative to the size of the other. For example, if a cake recipe calls for 2 eggs, 2 cakes would require 4 eggs, and so on. The ratio is 2:1 (2 eggs to 1 cake).

rational numbers: Numbers that can be expressed as a fraction (or ratio) using two integers. The decimal representation of a rational number will either terminate (end) or repeat (.55555…). For example, $^3/_4$ is a rational number, 2 is a rational number since it can be expressed as $^2/_1$, .625 (terminating decimal) is a rational number since it can be expressed as $^5/_8$, and .3333333333…(repeating decimal) is a rational number since it can be expressed as $^1/_3$.

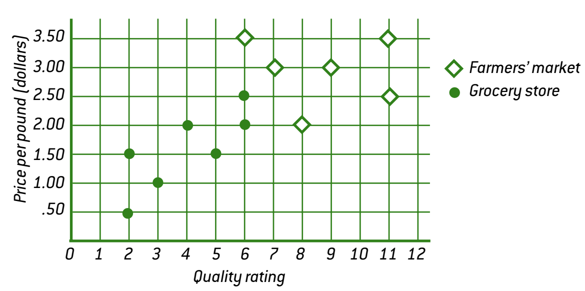
repeating decimal: A decimal number that continues without end in a repeating pattern, such as .33333… or .191919….

representation: The format for displaying a problem and/or its solution, such as a graph, a table, a diagram, a model, a picture, an equation, or a written description.

scale factor: The ratio or multiplier used to enlarge or reduce similar figures or shapes. For example, if a figure is $^1/_4$ the size of a similar figure, the scale factor is $^1/_4$, 25%, or .25. If it is 3 times bigger, the scale factor is 3 or 300%.

scatter plot or **graph:** A coordinate graph with points plotted to show a relationship between two corresponding sets of data, such as prices and quality ratings for different kinds of products.

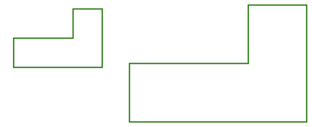
Fruit quality and price



scatter plot or graph

scientific notation: A form of writing very large or very small numbers using shortcuts such as exponents. For example, 75,000,000,000 could be written in scientific notation as 7.5×10^{10}

similar, similarity: Proportionally the same, but not necessarily the same size. Shapes are similar, or have similarity, when their corresponding sides are proportional and their corresponding angles are equal.


similar shapes

square: A rectangle with four congruent sides.

square number: A number that is the product of a whole number multiplied by itself. For example, 9 is a square number because 3x3 equals 9.

square root: One of two equal factors of a number. For example, 4 is the square root of 16, because 4x4 equals 16. Square roots are written using the $\sqrt{\ }$ symbol. For example, $\sqrt{9}=3$.

standard notation: The common way to write a number, without shortcuts such as exponents. For example, 4,300,000 and 34 are numbers written in standard notation.

stem-and-leaf plot: A chart that shows how data are grouped by place value. The "stem" shows the larger place values and the "leaves" show the digits with smaller place values.

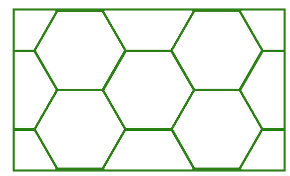
Temperature ranges
November – January

Stem	Leaf
2	1, 2, 3, 3, 4, 5, 6, 7, 9
3	1, 2, 3, 3, 4, 5, 5, 6
4	1, 2, 3, 5

stem and leaf plot
(stems represent tens, leaves represent ones)
Key: 3|5 means 35°

terminating decimal: A decimal number that ends (.25, .375), unlike repeating and nonrepeating decimals which continue without end.

tessellate, tessellation: Fit together in a pattern with no overlap or unused space. A honeycomb pattern is a tessellation—its hexagons tessellate.


tessellation

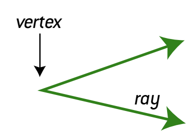
tiling: *See tessellate*.

transformation: The moving of a figure by sliding it, turning it, or flipping it.

unit rate: The rate for a single unit of the quantity being measured. For example, 55 mph means 55 miles per one hour—"55" is the unit rate.

variable: A symbol that represents a quantity that can change and/or an unknown quantity. In an equation that describes a changing situation, such as the relationship between profit and costs or time and distance, variables may be represented by letters—d (distance) = $10t$ (time, in hours), for example.

vertex: The point where two or more lines, rays, or line segments meet (intersect) to form an angle.



vertices: Plural of vertex.

***x*:** A letter commonly used to represent an unknown or changing number.

x axis: The horizontal number line on a coordinate graph.

***y*:** A letter commonly used to represent an unknown or changing number.

y axis: The vertical number line on a coordinate graph.

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